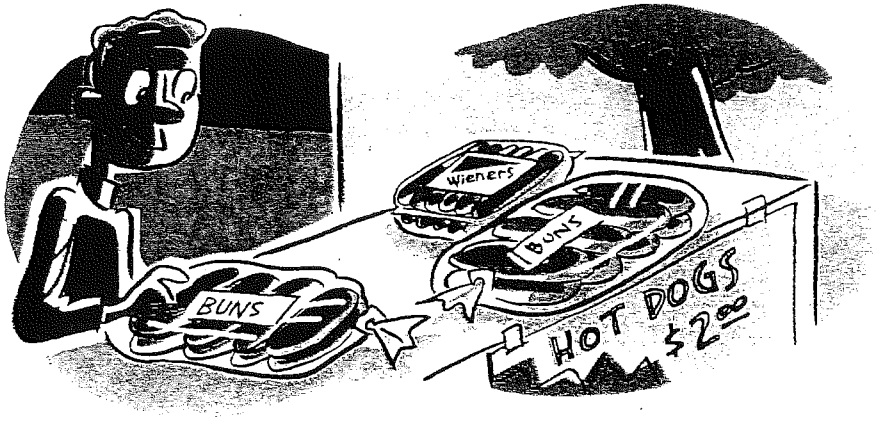
1.6 Determining Common Multiples

GOAL

Identify multiples, common multiples, and least common multiples of whole numbers.

LEARN ABOUT the Math

Denis's school is planning a hot dog lunch to raise money for sports equipment. Based on the last hot dog lunch, the students expect to sell at least 100 hot dogs. Wieners come in packages of 12, and buns come in packages of 8.



? How many packages of wieners and buns should the students buy to ensure that the numbers of wieners and buns are equal?

A. Calculate the possible sums of wieners by listing multiples of 12.

12, 24, 36, ...

B. Calculate the possible sums of buns by listing multiples of 8.

8, 16, 24, ...

common multiple

a number that is a multiple of two or more given numbers; for example, 12, 24, and 36 are common multiples of 4 and 6

least common multiple (LCM)

the least whole number that has two or more given whole numbers as factors; for example, 12 is the least common multiple of 4 and 6

C. Circle the numbers that are common to the lists in parts A and B. These are **common multiples** of 8 and 12.

D. Write the least of the circled multiples. This is the **least common multiple**, or **LCM**.

E. How can you use the LCM you identified in part D to determine other common multiples of 8 and 12?

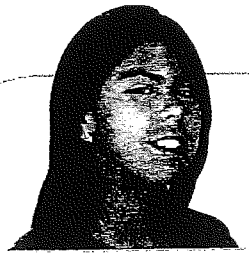
F. Explain how your answer in part E helps you solve the problem of how many packages of wieners and buns to buy.

Reflecting

G. Why do you think the problem would be easier to solve if buns came in packages of 6 and wieners came in packages of 12?

H. How can you tell if a number is a common multiple of two numbers? Use an example to help you explain.

WORK WITH the Math



Example 1 Determining common multiples

What is the LCM of 10 and 12? Identify three other common multiples.

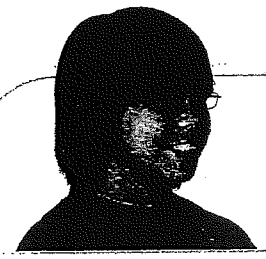
Oshana's Solution

10, 20, 30, 40, 50, (60)
The LCM of 10 and 12 is 60.

I listed the multiples of 10 first because it's easy to count by 10s. I stopped at the first number that was divisible by 12.

2 x 60 = 120
3 x 60 = 180
4 x 60 = 240
120, 180, and 240 are three other common multiples of 10 and 12.

Any multiple of the LCM, 60, is also a common multiple of 10 and 12.



Example 2 | Verifying a common multiple

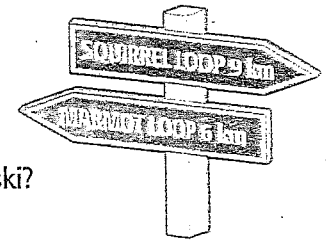
Is 31 620 a common multiple of 3, 4, and 5?

Denis's Solution

$3 + 1 + 6 + 2 + 0 = 12$	I used divisibility rules. The sum of the digits is divisible by 3, so 3 is a factor of 31 620.
31 620	4 is a factor of 20, so 4 is also a factor of 31 620.
31 620	The last digit is 0, so 5 is a factor of 31 620.
	3, 4, and 5 are all factors, so 31 620 is a common multiple of 3, 4, and 5.

Example 3 Solving a problem using common multiples

One afternoon, Star skied the Squirrel Loop cross-country trail and her mother skied the Marmot Loop trail. If they travelled the same distance, how many complete loops did each of them ski?



Sarah's Solution

Star skied 9 km in each loop, so her total distance is a multiple of 9 km. Her mother skied 6 km in each loop, so her mother's total distance is a multiple of 6 km. I need to find a common multiple of 6 and 9.

6, 12, (18) 18 is the LCM of 6 and 9.	I listed multiples of 6 and stopped at the first number that was divisible by 9. The next common multiple is $2 \times 18 = 36$, but I think 36 km is too far to ski in an afternoon.
$18 \div 6 = 3$ loops $18 \div 9 = 2$ loops	I divided 18 by each distance to determine the number of loops.
Star skied two loops, and her mother skied three loops.	

NEL

A Checking

1. List the first five multiples of each number.
   a) 2 **b)** 5 **c)** 6

2. Determine the LCM of 2, 5, and 6 by continuing the patterns in question 1.

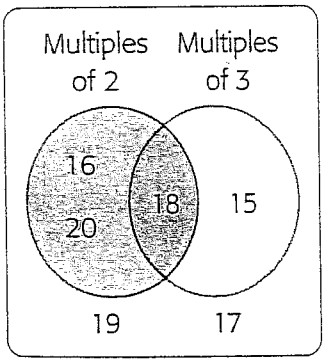
3. Show that 67 440 is a common multiple of 3 and 5 using divisibility rules.

B Practising

4. Determine the LCM of each set of numbers. Show your work.
   a) 9, 12 **b)** 3, 4, 6 **c)** 6, 10 **d)** 2, 3, 5, 20

5. How many packages of buns and soy patties should the school buy to sell in the cafeteria each week?
   - Buns are sold in packages of 6.
   - Soy patties are sold in packages of 8.
   - The school expects to sell between 80 and 100 soy burgers.
   - They want equal numbers of buns and burgers.

6. Identify the numbers that are common multiples of 5 and 8 using divisibility rules. Show your work.
   a) 195 **b)** 10 000 **c)** 13 731 **d)** 10 018



7. **a)** Complete the Venn diagram for the numbers 15 to 30.
   b) How can you describe the numbers that belong in the overlap?

8. Stephen is training for a triathlon. He runs every second day, swims every third day, and cycles every fifth day. How many times during the month of April will he practise all three events on the same day?

9. Max wrote an entry about the LCM in his math portfolio. Do you think his method is always correct? Use an example to help you explain.

> You can calculate the LCM of any two numbers by multiplying them together. For example, 2 x 3 = 6, so 6 is the LCM of 2 and 3.